

EDITORIAL
“INTERNATIONAL CONFERENCE ON VARIATIONAL ANALYSIS AND
NONSMOOTH OPTIMIZATION”
IN HONOR OF THE 65TH BIRTHDAY OF CHRISTIANE TAMMER (MARTIN
LUTHER UNIVERSITY HALLE-WITTENBERG)

This special issue covers the “International Conference on Variational Analysis and Non-smooth Optimization,” held online on July 15 - July 16, 2021. The mission of the conference was to bring together recognized experts as well as early-career researchers from countries across the globe to exchange the latest insights and present the recent scientific findings related to nonlinear and variational analysis, nonsmooth, vector, and set optimization, control theory, operations research, and related disciplines. A wealth of applications in economics, management science, engineering, mechanics, and behavioral sciences has led to the emergence of these theories. As a result of these important applications, these topics represent thriving research areas and branches of applied mathematics that continue to expand. This conference explored new developments, fostered new ideas, and encouraged participants’ collaborations to generate new knowledge that can be applied to existing problems and future applications.

Details on the conference, along with a brief description of the remarkable accomplishments of Professor Christiane Tamm, including laudations and conference photographs, can be found here: https://wiki.math.ntnu.no/_media/icvano2021/icvano_2021_booklet.pdf

This special issue is comprised of eleven articles whose contributions are as follows.

The paper “A unified approach to Bishop-Phelps and scalarizing functionals” by J. Jahn is devoted to a thorough and unifying investigation of newly proposed and existing scalarizing functionals associated with certain ordering cones. The proposed notion subsumes many such functionals, including the celebrated Bishop-Phelps functionals. It is shown that the studied scalarizing functionals provide a convenient framework for exploring fixed and variable order structures. The key advantages of the proposed approach are illustrated by means of numerous examples. Interesting applications to vector optimization and set optimization are also supplied.

The objective of B. Mordukhovich, and O. Nguyen in the paper “Subdifferential Calculus for Ordered Multifunctions with Applications to Set-Valued Optimization” is to study subdifferentials for set-valued maps taking values in certain ordered spaces. Two subdifferentials, basic and singular, are introduced, and new calculus rules are established. The calculus rules

are then applied for set-valued optimization problems, focusing on Pareto-type optimal solutions. Efficient conditions of the subdifferential Palais-Smale type are supplied, which ensure the existence of global Pareto-type minimizers in problems with geometric constraints. Novel existence results for set-valued optimization problems with functional constraints of equality and inequality types, as well as for operator and equilibrium types, are also given.

The aim of M. Durea in the paper entitled "Cone enlargements and applications to vector optimization" is to explore various types of conic enlargements. Valuable features of the cone enlargements and various relationships among them are studied. The behavior of cone enlargements in the context of Gerstewitz functional is explored. Exciting applications of cone enlargements to vector optimization are given.

G. Eichfelder, T. Gerlach, S. Rocktäschel, in the paper "Convexity and continuity of specific set-valued maps and their extremal value functions," conduct a thorough study of various classes of set-valued maps and the associated extremum value functions. The main results include recalling and establishing certain topological properties of the considered maps and statements concerning the associated extremum value functions. The main contribution is convexity notions for the set-valued maps as well as their continuity properties. The concepts of Lipschitz continuity also hold a special place in the discussion. The manuscript contains many examples illustrating the usefulness of the proposed notions and their limitations.

M. Jennane, E. M. Kalmoun, and L. Lafhim, in the paper "On nonsmooth multiobjective semi-infinite programming with switching constraints," investigate a nonsmooth multiobjective semi-infinite programming problem subject to the so-called switching constraints. In recent years, optimization problems with switching constraints (or similar notions) have been extensively studied because of their practical applications and interesting mathematical structure. The authors provide new optimality conditions and duality arguments for the considered program.

The focus of the paper "The stability of the parametric Cauchy problem of initial-value ordinary differential equations revisited," written by M. A. Mansour, J. Lahrache, and A. E. Ayoubi, is on giving new results concerning the quantitative stability of the approximate solutions to perturbed initial-value problems of ordinary differential equations. Firstly, considering parametric perturbations, a stability theory is given, which also permits deriving a valuable qualitative result. An interesting application of the main results to linear control systems is given. One of the main contributions of the paper is the study of the stability of approximate solutions. Qualitative and quantitative stability results for the sets of approximate solutions to the Cauchy problem are also supplied. The presented numerical results are in complete agreement with the theoretical findings.

The paper "The fixed point property of quasi-point-separable topological vector spaces" by J. Li deals with a new notion of a quasi-point-separable topological vector space. It is shown that the proposed class is quite large and subsumes the locally convex topological vector spaces and the pseudo-norm adjoint topological vector spaces as a particular case. An important contribution of this manuscript is that every quasi-point-separable Hausdorff topological vector space possesses the fixed point property. Employing nontrivial examples, it is shown that the class of quasi-point-separable topological vector spaces is strictly larger than the locally convex topological vector spaces. As a consequence, the established fixed point property extends the celebrated Tychonoff's fixed point theorem on locally convex topological vector spaces.

In the exciting contribution entitled "On the lower semicontinuity and subdifferentiability of the value function for conic linear programming problems," utilizing examples, C. Zălinescu shows that a technical lemma in the paper "N.E. Gretskey, J.M. Ostroy, W.R. Zame, Subdifferentiability, and the duality gap, *Positivity* 6: 261–274, 2002" is incorrect. Several other sentences and results of this paper are also critically analyzed.

The article entitled "Lagrange Duality of Vector Variational Inequalities" by X. Yang is devoted to studying vector variational inequalities with different constraints. The key idea is to employ a scalarization technique to convert the vector variational inequality into a scalar variational inequality, which is then transformed into an equivalent saddle point formulation. The focus is on vector variational inequalities with general and linear conic constraints. Numerous examples are given to illustrate the key advantages and limitations of the proposed approach.

P. Kumar and Khushboo, in the paper "Some topological properties of solution sets in partially ordered set optimization," study set optimization problems endowed with partial set order relations. The main contribution is deriving new results related to the continuity, generalized convexity, and arcwise connectedness of a recently introduced nonlinear scalarization function. For a family of scalarized optimization problems, new characterizations of the set of strict efficient and weak efficient solutions are also derived. Analogs of similar results are given under weaker convexity assumptions. An interesting application to an uncertain vector-valued game theory is also presented.

J. E. Martínez-Legaz, in the paper "Fenchel Subdifferential Operators: A Characterization without Cyclic Monotonicity," studies certain subdifferential characterizations. To be precise, inspired by the fact that Fenchel subdifferential operators of lower semicontinuous proper convex functions on real Banach spaces are characterized as maximally cyclically monotone or, equivalently, maximally monotone and cyclically monotone operators, this paper presents an alternative characterization, which does not involve cyclic monotonicity.

In conclusion, we express our sincere appreciation to the participants of the conference and the authors who contributed to this special issue.

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