

EDITORIAL
A SPECIAL ISSUE ON PROJECTION AND SPLITTING METHODS AND THEIR APPLICATIONS

Many mathematical formulations of significant real-world technological or physical problems are based on optimization with objective functions being the sum of several functions, each with different properties. This structure allows to employ *splitting methods* in which the problem is split into parts that can be treated more efficiently than the full problem via gradients, projections, proximal evaluations, and resolvents. Different splitting methods—such as the alternating direction method of multipliers (ADMM), the Douglas-Rachford method, and forward-backward splitting—iteratively combine such evaluations in different ways to solve the original problem.

This special issue brings together ten articles on recent advances in projection and splitting methods. The results present new approaches to challenging mathematical problems and applications in various fields such as variational inequalities, fixed point theory, and inverse problems. A summary of these articles in alphabetical order by first author is presented next.

H.H. Bauschke, H. Ouyang and X. Wang, in *On angles between convex cones*, present the extensions of results relating the Dixmier and Friedrichs angles associated with pairs of linear subspaces to the special case where those subspaces are convex cones. These angles are an essential tool in analyzing the convergence rates of projection-based algorithms.

S. Deyo and V. Elser, in *Avoiding traps in nonconvex problems*, provide the guidelines on the tuning of parameters for iterative projection algorithms when they are applied to nonconvex problems. The importance of tuning both hyperparameters and metric parameters is discussed, and illustrated through examples.

A. Gibali and Y.I. Suleiman, in *Parallel projection method for solving split equilibrium problems*, introduce a new class of *bilevel multiple sets split equilibrium problems* and present a parallel projection method for solving them. A proof of convergence and numerical illustration is also provided.

B. Peters, in *Point-to-set distance functions for output-constrained neural networks*, develops a new algorithm to constrain the output of neural networks for semantic segmentation based on prior information. The method achieves this through the use of projection-based point-to-set distance functions. The effectiveness of the method is highlighted with relevant examples from image processing and geosciences.

H. Rehman, M. Özdemir, İ. Karahan and N. Wairojjana, in *The Tseng's extragradient method for semistrictly quasimonotone variational inequalities*, study a weakly convergent method for solving variational inequalities with semistrictly quasimonotone and Lipschitz-continuous mappings.

S. Reich and A.J. Zaslavski, in *Convergence of inexact iterates of strict contractions in metric spaces with graphs*, present an analog of Butnariu et al. result from 2016 in which it is shown that if all exact orbits of a nonexpansive mapping converge to a fixed point, then this convergence property also holds for all its inexact orbits with summable errors.

B. Tan and X. Qin, in *Adaptive modified inertial projection and contraction methods for pseudomonotone variational inequalities*, present four modified inertial projection and contraction algorithms with non-monotonic step sizes for solving variational inequalities.

W. Yata, M. Yamagishi and I. Yamada, in *A constrained LiGME model and its proximal splitting algorithm under overall convexity condition*, propose a constrained LiGME (cLiGME) model for solving convex optimization where the convex constraint sets express a priori knowledge regarding a certain unknown vector to be estimated. Moreover, a proximal splitting algorithm is introduced and analysed for the cLiGME model.

J. Ye, C. Wan and S. W. Fung, in *Adaptive uncertainty-weighted ADMM for distributed optimization*, introduce an adaptive uncertainty-weighted consensus ADMM method for solving large-scale convex optimization problems in a distributed manner.

A.J. Zaslavski, in *Superiorization with a projected subgradient method*, studies a constrained minimization problem with a convex objective function and with a feasible region that is the intersection of finitely many closed convex constraint sets. A projected subgradient method combined with a dynamic string-averaging projection method is introduced and analysed.

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